

FMF



Exercise Booklet



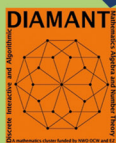
LIMO 2018



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Tips and tricks

During the competition the following rules apply:

- Write your solutions for each problem on a separate sheet, and write down your team name, your team number and the problem number on every sheet.
- Facilities such as books, graphing calculators, mobile phones and laptops are not allowed. Non-graphing calculators may be used. You may only communicate with teammates and with the organization.
- Water is provided during the competition. For any questions, there will be a member of the organization present in the competition room.

Tips that may help you during the competition:

- Notation. The notation and/or the terminology will be explained for certain problems. Furthermore, with the natural numbers the collection $\{1, 2, 3, \dots\}$ is meant, which we denote with \mathbb{N} .
- Order of difficulty. The problems are sorted in increasing difficulty, with easiest first. Each problem is worth 10 points.
- Read the problems carefully. If you start solving too early, you may have overlooked important information. Sometimes a problem contains a (hidden) hint that indicates what strategy to use.
- Be a team. Divide the tasks, but ask each other for help if needed. Consider where everyone's qualities lie, and check each other's work during the match; maybe there are still mistakes.
- Try to collect at least some points. A partial solution may still earn you points. If you do not have the solution for a specific sub-problem, you may still use the result for all subsequent sub-problems.
- Find a pattern. For example, if you have to prove something for all $n \in \mathbb{N}$, try to see what happens for $n = 1$ or $n = 2$. Discover a pattern and prove that this pattern continues.
- "Houd het gezellig" is Dutch for enjoy yourself and be nice to others.

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This Exercise Booklet is a product of the LIMO-committee 2018:

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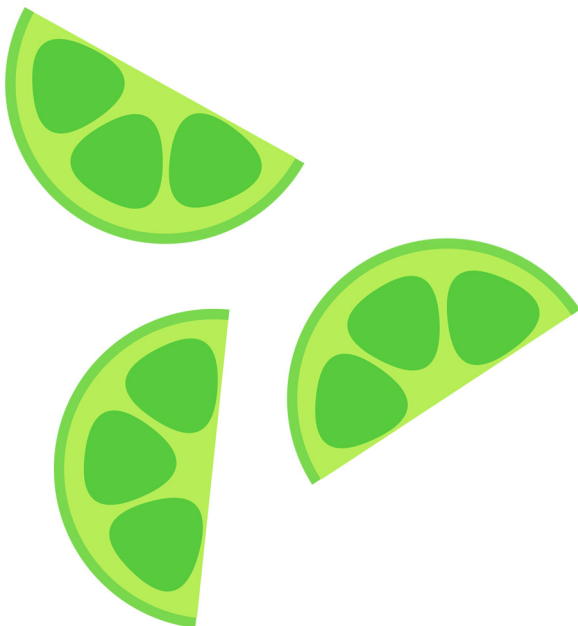
Website: fmf.limo.nl

1. Even triangles are odd

Harry Smit MSc., Universiteit Utrecht

Suppose you have a triangle that is subdivided into other triangles, i.e. this triangle is completely tiled with smaller triangles, without any overlap. Furthermore, assume that no three vertices (not even those of different triangles) lie on a straight line.

- (a) Let k be an odd positive integer. Prove that it is possible to make a subdivision into exactly k triangles.
- (b) Let k be an even positive integer. Prove that it is impossible to make a subdivision into exactly k triangles.





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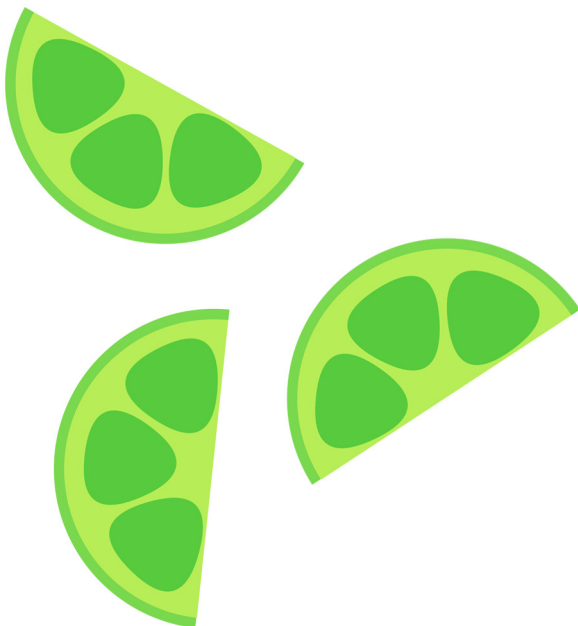
2. Self-divisible numbers

Ir. Harold de Boer, Transtrend BV

We define a positive integer to be self-divisible when each digit of the number (starting from the second digit) is a divisor of the number formed by the digits preceding it. For example, 42771 is self-divisible because 4 is a multiple of 2, 42 is a multiple of 7, 427 is a multiple of 7 and 4277 is a multiple of 1.

We define G_N as the set of self-divisible numbers that consist of N different digits. So, 42771 is an element of the set G_4 .

- (a) Determine the largest N for which the set G_N is not empty.
- (b) Determine the smallest number in this particular set G_N .



3. Ramification

Prof. dr. Hendrik Lenstra, Universiteit Leiden

A *tree* is a finite connected graph without cycles. An *automorphism* of a graph is a map σ that permutes both the set of vertices and the set of edges of the graph, in such a way that an edge e connects two vertices v and w if and only if $\sigma(e)$ connects $\sigma(v)$ and $\sigma(w)$.

Prove that every tree has a vertex or an edge that is mapped to itself by every automorphism of the tree.





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4. A Kurt Mahler¹ style power series

Prof. dr. Gunther Cornelissen, Universiteit Utrecht

Fix a prime number p . If an integer n is written as $n = p^r u$ where u is coprime to p , we define $|n|_p := p^{-r}$. Consider the power series

$$f(z) := \sum_{n \geq 1} |n|_p z^n.$$

(a) Show that the power series defining $f(z)$ converges for all complex numbers z with $|z| < 1$.

(b) Prove that for $|z| < 1$,

$$f(z) = \frac{z}{1-z} - \frac{z^p}{1-z^p} + \frac{1}{p} f(z^p).$$

(c) Prove that the series defining $f(z)$ diverges at a dense set of points in the complex unit circle by showing that

- (i) it diverges at all p -power roots of unity, i.e., at all $z \in \mu_{p^\infty} := \{e^{2\pi ki/p^r} : k, r \in \mathbf{Z}\}$;
- (ii) μ_{p^∞} is dense in the unit circle (i.e, for every $z_0 \in \{|z| = 1\}$ and $\varepsilon > 0$ there exists $\zeta \in \mu_{p^\infty}$ with $d(z_0, \zeta) < \varepsilon$ where d is distance along the circle).



¹The famous number theorist Kurt Mahler (1903-1988) studied these kinds of problems in the 1930's. Between 1934 and 1936 he worked - after from escaping Germany via Manchester - in Groningen, where he had a bicycle accident, after which he walked with a limp for the rest of his life.

5. Chebyshev polynomials

Prof. dr. Jaap Top, Rijksuniversiteit Groningen

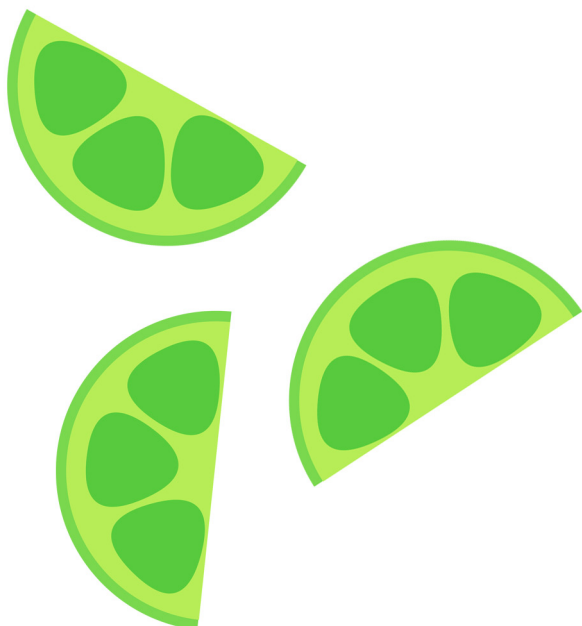
The wikipedia page describing ‘Chebyshev polynomials’ briefly mentions, besides the usual Chebyshev polynomials of the first kind T_n and the ones of the second kind U_n also a sequence of polynomials C_n . The sets of polynomials are related by $C_n(x) = 2T_n(\frac{x}{2})$ and $T'_n = nU_{n-1}$. This problem defines and asks for some properties of the polynomials C_n .

- (a) Show that for any integer $n \geq 1$ a *unique* real polynomial C_n in one variable exists such that

$$x^n + x^{-n} = C_n(x + x^{-1})$$

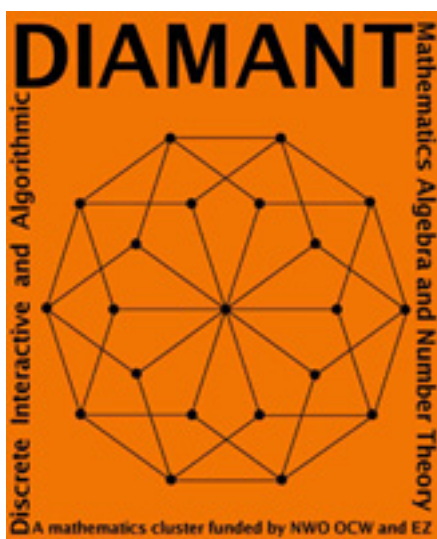
(here x is also a variable).

- (b) Show that $(4 - t^2)C_n''(t) - tC_n'(t) + n^2C_n(t) = 0$.
- (c) Show that for every $n \geq 2$ the polynomial $C_n \bmod 2$ (which has coefficients in $\mathbb{Z}/2\mathbb{Z}$) can be factored as a product $f^2 \cdot g$ for polynomials f, g with coefficients in $\mathbb{Z}/2\mathbb{Z}$ with moreover f of degree at least 1.



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Katholieke Universiteit Leuven

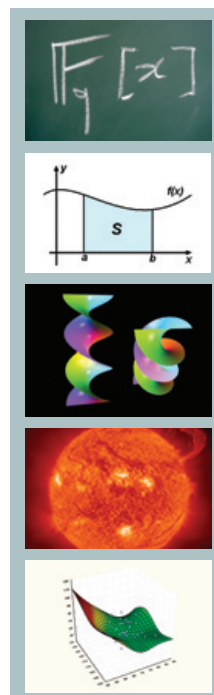
De KU Leuven, gesticht in 1425, is de oudste universiteit van de lage landen. Meer dan 4.500 onderzoekers zijn er actief in wetenschappelijk onderzoek en onderwijs. Op 5 februari 2018 telde de KU Leuven in totaal 56.649 ingeschreven studenten. Van deze studenten heeft ongeveer 84% de Belgische nationaliteit, 16% zijn internationale studenten. Dit maakt van de gezellige provincie-hoofdstad Leuven een bruisende studentenstad met een rijk sociocultureel aanbod.

Onderzoek aan het Departement Wiskunde

Het onderzoek aan het departement Wiskunde is georganiseerd op het niveau van de onderzoeksafdelingen:

- **Afdeling Algebra:** het onderzoek situeert zich in de algebraïsche meetkunde, getaltheorie, algebraïsche topologie en groepentheorie.
- **Afdeling Analyse:** in deze afdeling doet men onderzoek in de klassieke analyse (reële en complexe analyse) en in de functionaalanalyse.
- **Afdeling Meetkunde:** het onderzoek is gecentreerd rond differentiaalmeetkunde, in het bijzonder Riemannse en pseudo-Riemannse meetkunde en deelvariëteiten.
- **Afdeling Plasma-astrofysica:** het onderzoeksdomein van deze afdeling is de wiskunde van vloeistoffen en plasma's, het voornaamste studieobject is de zon. Dit onderzoek is gesitueerd in de toegepaste en computationele wiskunde.
- **Afdeling Statistiek:** deze afdeling is actief in de wiskundige statistiek, in het bijzonder de theorie van extreme waarden, robuuste statistiek en niet-parametrische methoden. Ook stochastische processen en financiële wiskunde komen aan bod. De afdeling is bovendien ook actief in toegepaste consultatie voor bedrijven.

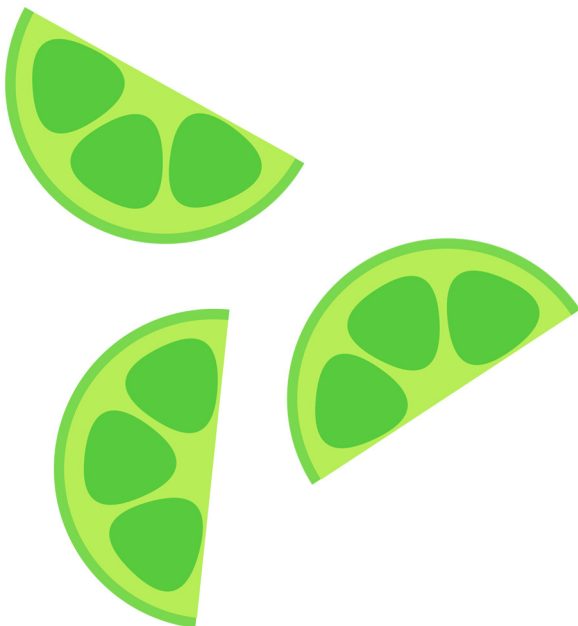
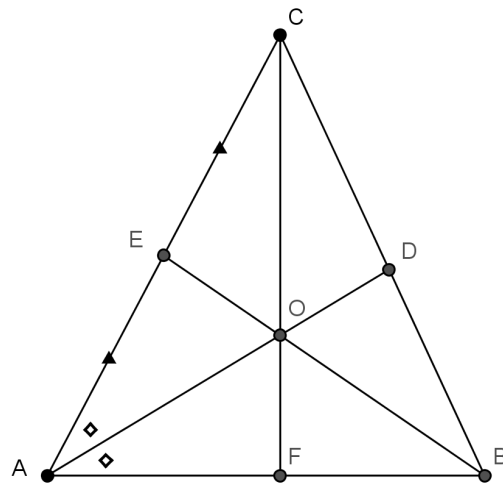
Meer info op <http://wis.kuleuven.be>



6. Medium competition in bisecting the altitude

Eduardo Ruíz Duarte MSc., Rijksuniversiteit Groningen

Find an example of a non-equilateral triangle ABC with integer sides for which the altitude from A , the bisector at B , and the median at C are concurrent, that is, they intersect at a single point. Hint: it is possible to find such a triangle with a perimeter no greater than 50.



7. Mathematicians like to play with colors

Stijn Cambie MSc., Radboud Universiteit Nijmegen

A *finite coloring* of the nonnegative integers is a map $f: \mathbb{N} \mapsto S : n \mapsto f(n)$, where S is a finite set. Given a coloring f , a set X is called *monochromatic* (MC) when every element is colored in the same color, i.e. for all $x_1, x_2 \in X$ one has $f(x_1) = f(x_2)$.

A theorem of Schur says that any finite coloring of \mathbb{N} contains a MC set $\{x, y, x + y\}$, where $x, y \in \mathbb{N}$.

- (a) Prove that any finite coloring of \mathbb{N} contains a MC set $\{a, b, ab\}$ as well, where $a, b > 1$.
- (b) Given a coloring $f: \mathbb{N} \mapsto \{B, W\}$. Does there exist a MC set of the form $\{a, b, ab + a\}$?
- (c) Given a finite coloring of \mathbb{N} . Does there exist a MC set of the form $\{a, b, ab, a(b+2018)\}$?





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8. Typing a's and b's

Dr. Daniel Valesin, Rijksuniversiteit Groningen

I type letters at random in my keyboard. Each letter I type is equally likely to be any of the 26 (lowercase) letters of the alphabet, chosen independently of what has been typed before. I stop typing when I type the letter a three times in succession (that is, with no other letter in between). What is the expected number of times I type the letter b?

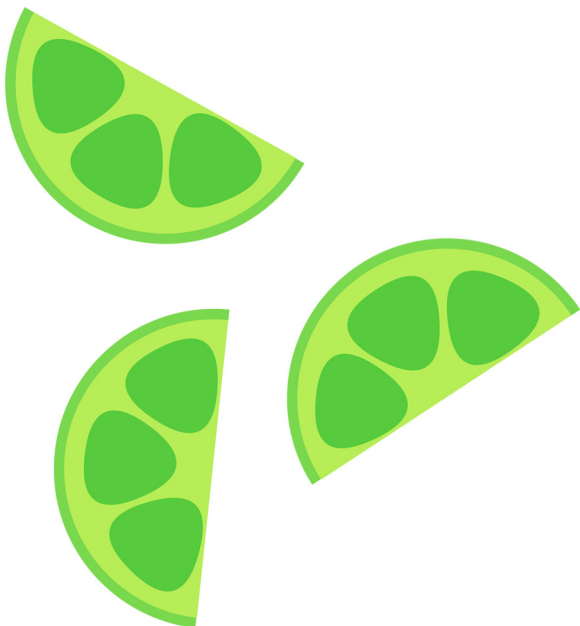


9. Matrix-valued function in one variable

Leslie Molag MSc., Katholieke Universiteit Leuven

Let $f : \mathbb{R} \rightarrow \mathbb{R}^{3 \times 3}$ be a C^∞ function such that $f(0) = \mathbb{I}$ and $\det f(x)$ is constant.

- (a) Prove that $f'(0)^3 = 0$ when f is a linear function, i.e. when $f(x) = \mathbb{I} + xf'(0)$.
- (b) Show that it is not guaranteed that $f'(0)^3 = 0$.
- (c) Is $f'(0)^3 = 0$ guaranteed under the additional requirement that $f''(0) = 0$?





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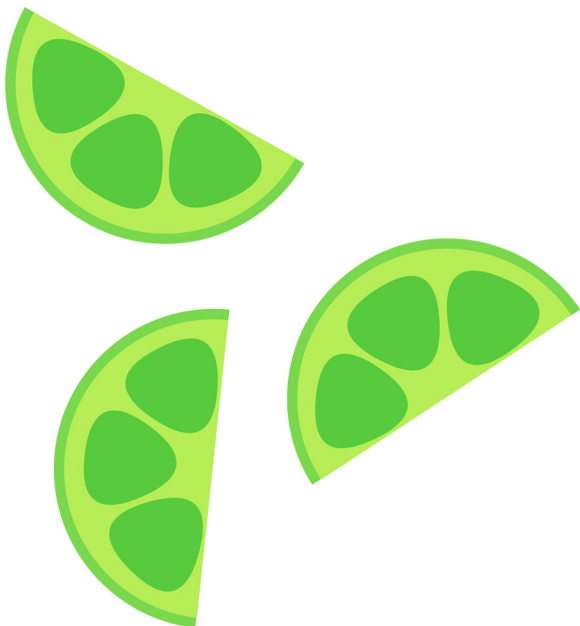
10. Rational points

Prof. dr. Frans Oort, Universiteit Utrecht

Consider a prime number $p \equiv 3 \pmod{4}$. Show:

$$\#\{(x, y) \in (\mathbb{F}_p)^2 \mid 1 = x^2 + y^2 + x^2y^2\} = p + 1;$$

we write $\mathbb{F}_p = \mathbb{Z}/p\mathbb{Z}$.



11. How about this question?

Prof. dr. Lex Schrijver, Universiteit van Amsterdam

Let L be a linear subspace of $(\mathbb{Z}/2\mathbb{Z})^n$. Call an element x of L *maximal* if the support of x is not contained in the support of any other element of L . Prove that the number of maximal elements of L is odd.



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 - You partake in the [Mastermath programme](#), meaning you can follow the best mathematics courses, regardless of the university in the Netherlands that offers them. It gives you the opportunity to interact with fellow mathematics students from all over the country.
 - As a Mathematics Master's student of certain specialisations you get to work closely with the mathematicians and physicists of the [Institute for Mathematics, Astrophysics and Particle Physics](#), as well as the computer scientists at the [Institute for Computing and Information Sciences](#).
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Je bent nu nog volop aan het genieten van je studententijd, maar weet jij al waar je straks jouw bèta-talent wilt inzetten? Bij Talent&Pro zijn we op zoek naar ambitieuze starters voor onze traineeships. Met jouw bèta-achtergrond ben je van grote waarde voor uitdagende opdrachten binnen de financiële dienstverlening.

Actuariel Rekenaar

Is werken met cijfers helemaal jouw ding? Los je graag onmogelijke vraagstukken op en ben je analytisch ingesteld? Dan ben je bij het traineeship Actuariel Rekenaar aan het juiste adres! Met ons opleidings- en ontwikkeltraject groei je versneld naar een rol als actuariel analist, actuaris, risk manager of data analist. In dit vakgebied pas jij jouw wiskundige inzicht toe op vraagstukken in het bedrijfsleven.

Business IT

Of ben je juist goed in het vertalen van de gebruikersbehoefte naar effectieve en betrouwbare IT-oplossingen? En kun jij ook de perfecte link leggen tussen gebruikers en programmeurs van informatiesystemen? Dan past het traineeship Business IT goed bij jou. Je wordt binnen no-time opgeleid tot data-, informatie- of businessanalist!

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Solliciteren?

Wil jij binnen korte tijd enorm veel werkervaring opdoen bij de grootste spelers zoals ABN AMRO, Delta Lloyd en Aegon? Zie jij jezelf al rondrijden in één van onze auto's? En wil jij na je studie aan de slag als Actuariel Rekenaar of werken aan de meest complexe IT-vraagstukken? Bekijk onze website en neem contact op met één van onze recruiters voor meer informatie of een kennismaking.

12. Restrict the extension

Prof. dr. Robert Tijdeman, Universiteit Leiden

Let $(v_n)_{n=0}^{\infty}$ be a sequence in \mathbb{Z}^2 with $v_0 = (1, 0)$, $v_1 = (0, 1)$ and for $n = 1, 2, \dots$

$$v_{n+1} = v_n + 2v_{n-1} \text{ or } v_{n+1} = v_n - 2v_{n-1}.$$

(a) Prove that for $n = 2, 3, \dots$ the set

$$V_n := \left\{ \sum_{i=0}^{n-1} \varepsilon_i v_i \text{ with } \varepsilon_i \in \{0, 1\} \text{ for } i = 0, 1, \dots, n-1 \right\}$$

consists of 2^n distinct points.

(b) Prove that the diameter of the convex hull of V_n for $n \geq 6$ is at least $(\sqrt{2})^n (> 1.414^n)$.

(c) Prove that the signs can be chosen in such a way that the diameter of the convex hull of V_n for $n = 2, 3, \dots$ is at most 1.67×1.601^n .

P.S. The convex hull W of V is the smallest set such that if $v, w \in V$, then the connecting line segment belongs to W .



Exercises made by:

Harry Smit MSc.

Ir. Harold de Boer

Prof. dr. Hendrik Lenstra

Prof. dr. Gunther Cornelissen

Prof. dr. Jaap Top

Eduardo Ruíz Duarte MSc.

Stijn Cambie MSc.

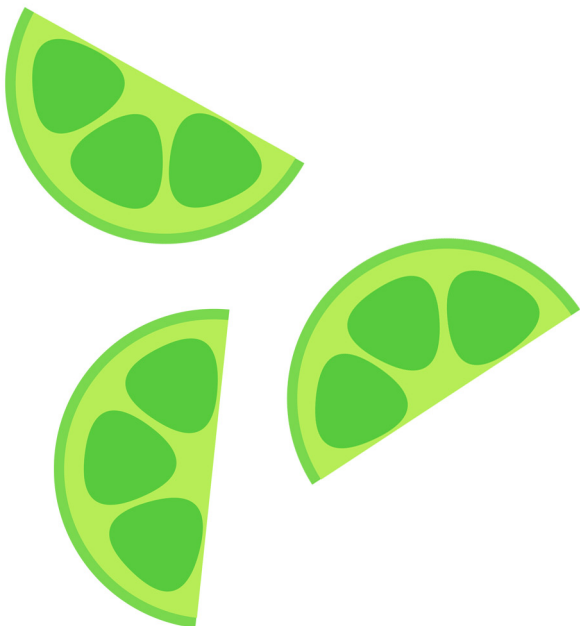
Dr. Daniel Valesin

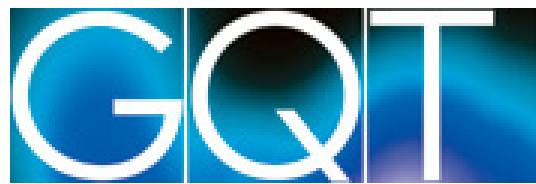
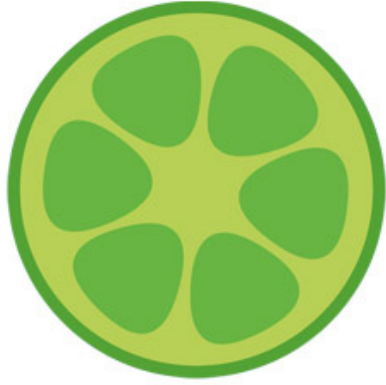
Leslie Molag MSc.

Prof. dr. Frans Oort

Prof. dr. Lex Schrijver

Prof. dr. Robert Tijdeman







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